Appendix A

Combination of track probabilities

In this appendix, the various event probabilities given in section 3.2.2 are derived. The product

$$\Pi = \prod_{i=1}^{n} P(|S_i|), \tag{A.1}$$

where P_i is the probability of track i to come from the primary vertex, can be used to determine which event is a B event and which is not. However, this value depends very much on the number of tracks in an event. It is better to use the probability, or actually the confidence level, that an event with no B would have a product of probabilities less than or equal to the observed value of Π, i.e. $C_n(\Pi) = (\text{prob.that}) \prod_{i=1}^{n} x_i < \Pi$, where x_i are uniformly distributed variables representing the possible values for a given track-probability and Π is the product of observed variables of x_i as defined in equation A.1.

Let us start with the simple two-track case. The situation is plotted in figure A.1. The confidence level for two tracks $C_2(P_1P_2)$ is just one minus the (shaded) area inside the hyperbola defined by the equation: $x_1 = P_1P_2/x_2$. This leads to

$$C_2(P_1P_2) = 1 - \int_{\Pi}^{1} \int_{\Pi/x_2}^{1} \delta x_1 \delta x_2 = \Pi\,(1 - \ln \Pi)\,. \tag{A.2}$$

One can extend the above example to more than two tracks, using the fact that

$$\int \frac{\ln^n x}{x} \mathrm{d}x = \frac{1}{n+1} \ln^{n+1} x. \tag{A.3}$$

For n tracks this leads to $C_n(\Pi) = \Pi(1 - \ln\Pi + \frac{1}{2}\ln^2\Pi - \cdots (-)^{n-1} \frac{1}{(n-1)!} \ln^{n-1}\Pi)$. This can be written in a simpler form as:

$$C_n(\Pi) = \Pi \left(\sum_{i=0}^{n-1} \frac{(-\ln\Pi)^i}{i!} \right). \tag{A.4}$$

A more mathematical derivation of this confidence level can be found in [65]. Since this event confidence level is a very small quantity in the case of B-meson decays, the region of interest is expanded by defining the discriminant B_n as: $B_n = -\log C_n$. The efficiency and purity as a function of this event discriminant is plotted in figure A.2

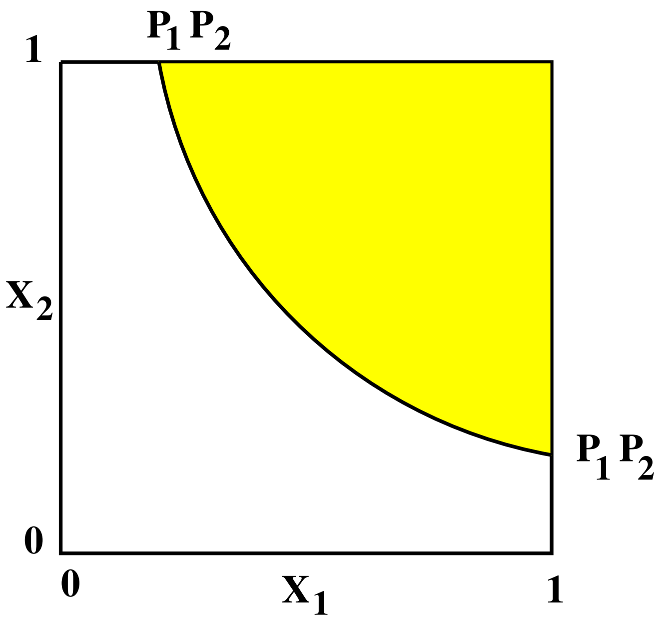

Figure A.1: *Determination of the total probability in the simple two-track case.*

In the confidence level obtained above only tracks with positively signed decay lengths are used. The idea behind this is that tracks with large negatively signed decay lengths cannot be B-decay particles. Low probabilities from the negative side would only add 'noise' to the calculation of the event probability. However, knowing the multiplicity can be helpful. This can be seen as follows: If there are more negatively signed tracks, then the confidence level for getting a few tracks with small positive decay lengths is higher. To account for such fluctuations, the probability for negative-decay-length tracks is defined to be unity (negatively-signed tracks come very probably from the primary vertex). In a u,d,s event about half of the tracks should have a negative sign. The confidence level obtained in this way can be written in terms of the previous confidence level obtained in equation A.4:

$$C_n' = \frac{1}{2^n}\sum_{j=1}^{n}\binom{n}{j}C_j = \frac{1}{2^n}\Pi\sum_{j=1}^{n}\binom{n}{j}\sum_{i=0}^{j-1}\frac{-\ln(\Pi)^i}{i!}. \tag{A.5}$$

The efficiency and purity as a function of the event discriminant is plotted in figure A.3. When compared to figure A.2, one can see that the latter confidence level gives better efficiency values for the same purity.

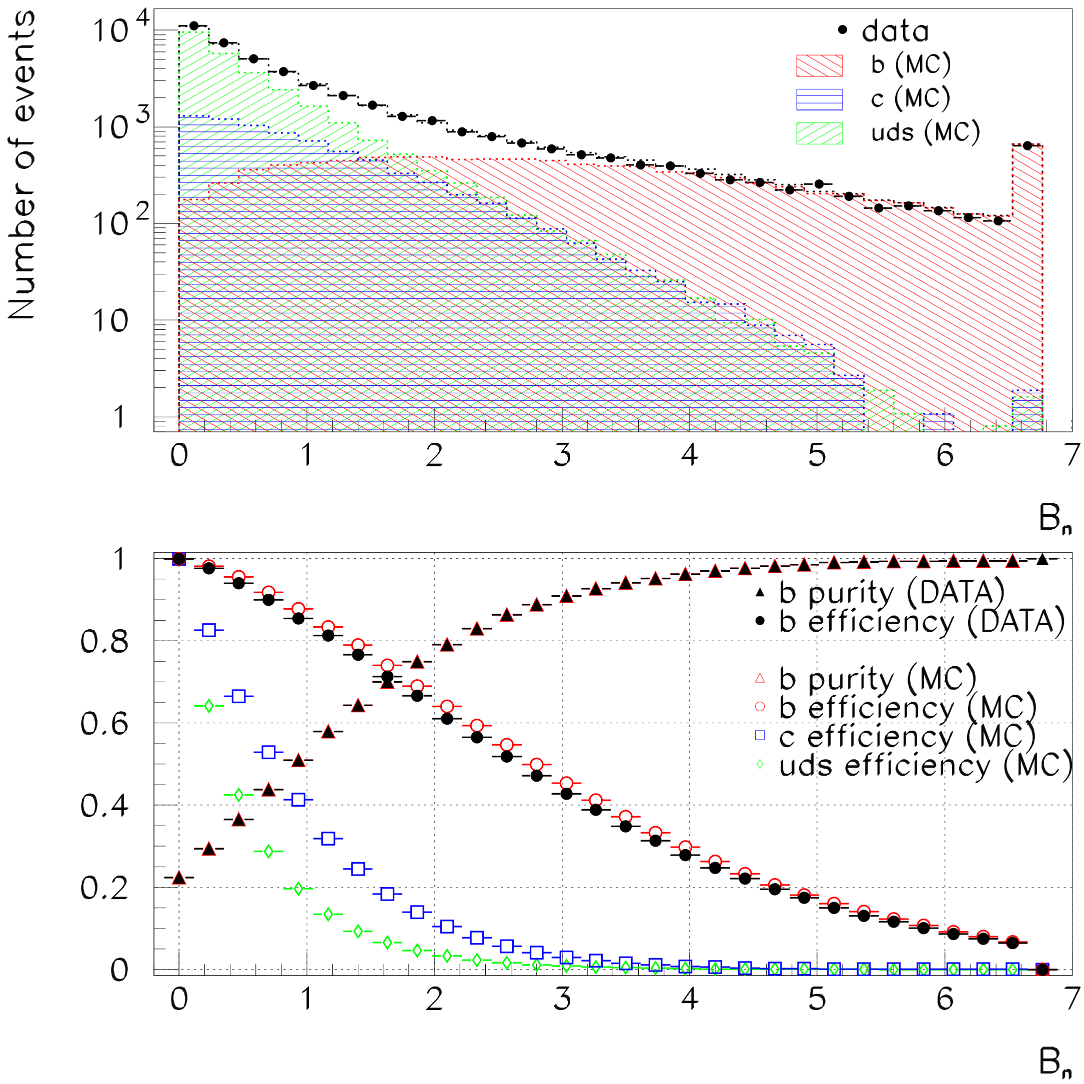

Figure A.2: *The event discriminant distribution is shown in the upper plot for the case that only positively signed tracks were taken into account. The last bin is the overflow bin. The efficiency and purity as function of a cut on the discriminant is shown in the bottom plot.*

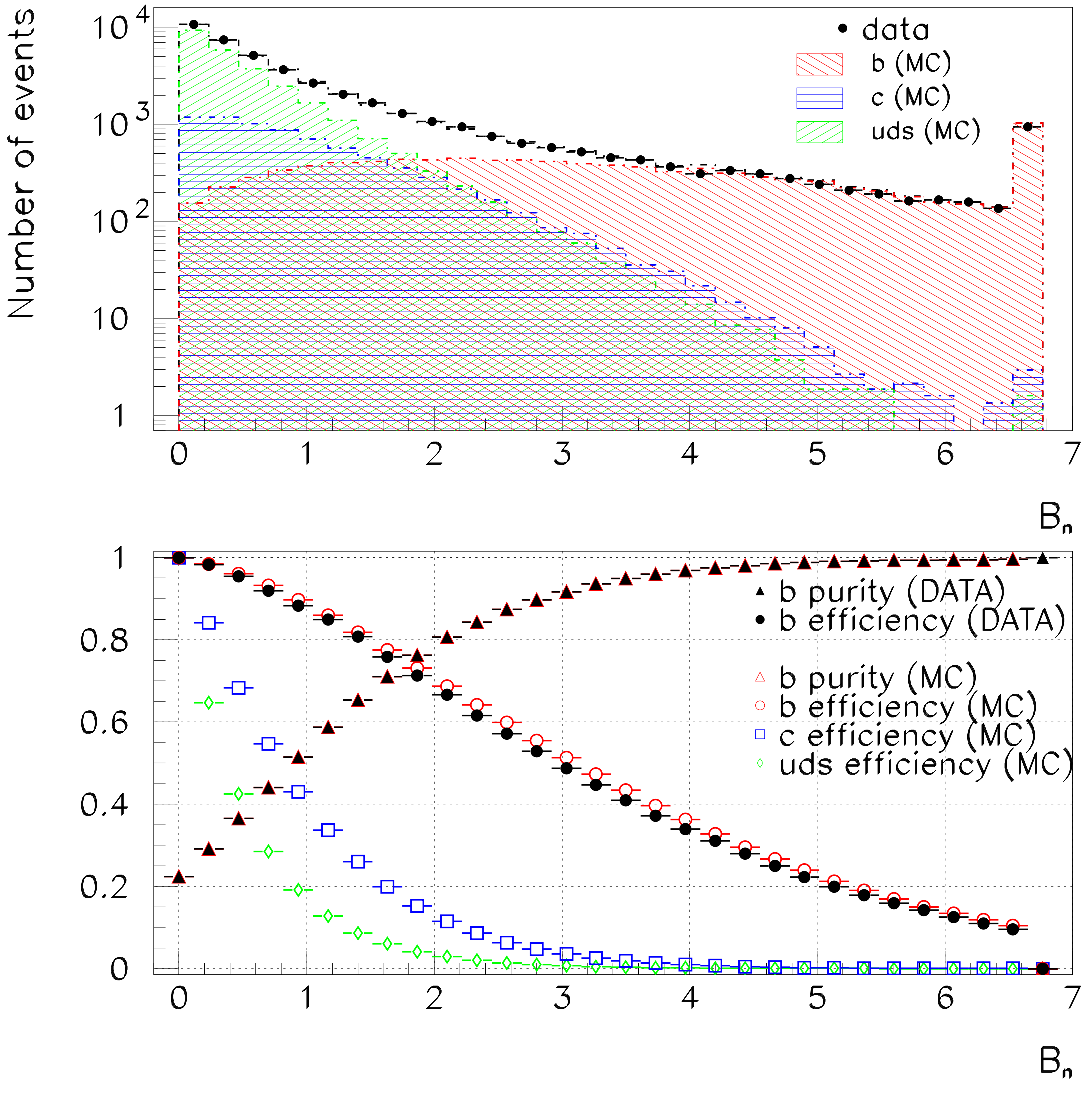

Figure A.3: *The same as figure A.2, but here also negatively signed tracks were taken into account for the calculation of the event discriminant.*